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PIV- and image-based flow analysis of a steady and pulsed jet using Dynamic Mode Decomposition

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ABSTRACT

Time-resolved particle-image velocimetry data of an axisymmetric jet have been used in conjunction with a novel data-analysis technique known as the Dynamic Mode Decomposition (DMD) to extract flow features that contribute significantly to the principal dynamics of the jet. In contrast to other decomposition techniques (such as POD), our study reveals coherent structures in the flow together with their frequency and growth/decay rates. Acoustic forcing has been applied, and the response behavior has been decomposed using DMD. Significant differences in the DMD-spectrum and the associated modal structures can be observed between the forced and unforced jet.

1. INTRODUCTION

The response of a flow to external excitation is a diagnostic tool that can provide a wealth of information to practitioners and theoreticians. Mathematically speaking, it provides the transfer function of the system by linking an input signal to an output response. By scanning over a range of frequencies, the linear behavior of a system (or the linearized approximation of it) can be extracted which provides first quantitative results on the sensitivity to external excitation but also first suggestions on how to best manipulate the flow.

In order to evaluate the response behavior of flows, it is necessary to have a diagnostic tool to quantify and decompose the fluid response. For numerical simulations of fluid flow this step does not pose a problem. The spectral evaluation of forced problems or, more recently, the computation of resolvent norms (i.e., transfer function norms) has become commonplace in the computational fluid dynamics community. Even for large-scale applications, efficient algorithms are readily available [4], even if they require iterative (Krylov subspace) techniques or model reduction efforts. All these techniques, however, require at some stage the knowledge of a discretized model, commonly in the form of the linearized fluid equations. Matrix-vector multiplications, using the Jacobian stability matrix and selected flow fields, are an integral part of the available algorithms (see [1]) and are responsible for their effectiveness and robustness.

For experimental settings, techniques that require an underlying model are not attractive; only data is available in this case. The question then arises if some or any of the above algorithms can be modified to extract pertinent information from the flow by using measurements only and by abandoning the necessity of an explicit model. The Dynamic Mode Decomposition [8, 7, 6] is a step in this new direction of "model-free" algorithms for quantitative flow analysis.

2. BACKGROUND

Rather than assuming a model (described by the system matrix $\mathbf{A}$ for a discrete formulation) for our flow situation, we focus on a set of measurements $\mathbf{v}$ which we gather in an equispaced temporal sequence of N snapshots, i.e.,

$$\mathbf{V}_1^N = \{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_N\}. \qquad (1)$$

We then proceed by assuming a constant (or quasi-constant) mapping from one snapshot to the next. This mapping shall be called $\mathbf{A}$ and is equivalent to the above-mentioned system matrix for a linear flow. We then know that

$$\mathbf{A}\mathbf{V}_1^{N-1} \approx \mathbf{V}_2^N \qquad (2)$$

which states that each snapshot evolves into the subsequent one under the action of the snapshot-to-snapshot mapping $\mathbf{A}$. To close this argument, we invoke the idea underlying the Arnoldi technique [9, 2] and express the right-hand side of (2) as a linear combination of the snapshot basis $\mathbf{V}_1^{N-1}$ which can be written as

$$\mathbf{A}\mathbf{V}_1^{N-1} \approx \mathbf{V}_1^{N-1}\mathbf{S}. \qquad (3)$$

It can easily be verified that the matrix $\mathbf{S}$ is of companion type [5]. In other words, it can be determined by expressing the last snapshot $\mathbf{v}_N$ by a linear combination of all previous snapshots $\mathbf{V}_1^{N-1}\mathbf{s}$ with $\mathbf{s}$ as the last column of the companion matrix $\mathbf{S}$.

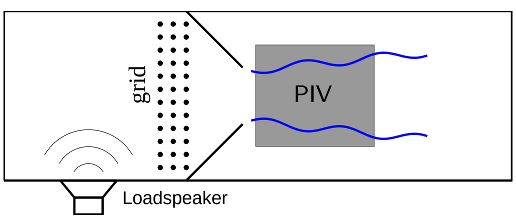
Once the matrix $\mathbf{S}$ has been identified from the data, it can be taken as a low-dimensional replacement of the system matrix $\mathbf{A}$. In particular, the stability or response behavior of our fluid system can now be extracted, at least in an approximate manner, from the matrix $\mathbf{S}$.

The reliance on data only for the extraction of dominant flow behavior by the dynamic mode decomposition has some disadvantages but, by and large, overwhelming advantages. First and foremost among the advantages is the applicability of DMD to experimentally gathered data. This fact introduces new and quantitative technology to the analysis of experimental data, promising new insight into fluid processes that can be better or efficiently measured than simulated. Image-based visualizations using passive dye tracers in conjunction with high-speed cameras can equally be processed [7] as time-resolved particle-image-velocimetry (PIV) data. In addition, the dissection of the entire flow field into subdomains where localized instability mechanisms prevail is possible [6], a feature that presents substantial challenges for numerical simulations. It allows the breakup of multiscale phenomena into their respective components. Furthermore, a distinction between temporal and spatial dynamics — again a difficult undertaking for computational investigations — is less pronounced for a "model-free" algorithm since for neither case a system matrix has to be formed; whether we perform a temporal or a spatial analysis thus only depends on the manner in which the measured data are processed. These significant advantages come at the expense of a (minor) degradation in convergence, as compared with the iterative algorithms that require a system matrix **A**. However, by a small modification of the above algorithm (see [6]) satisfactory convergence and robustness can be achieved. In this sense, the advantages and possibilities of the dynamic mode decomposition far outweigh the minor limitations in its convergence properties.

3. RESULTS

To demonstrate the capabilities of the dynamic mode decomposition we consider time-resolved particle-image-velocimetry (TR-PIV) measurements of an axisymmetric jet. In particular, two cases are considered: (i) the dynamics of the jet under no external forcing, and (ii) the response of the jet when subjected to acoustic excitation of a harmonic nature.

(a)



(b)

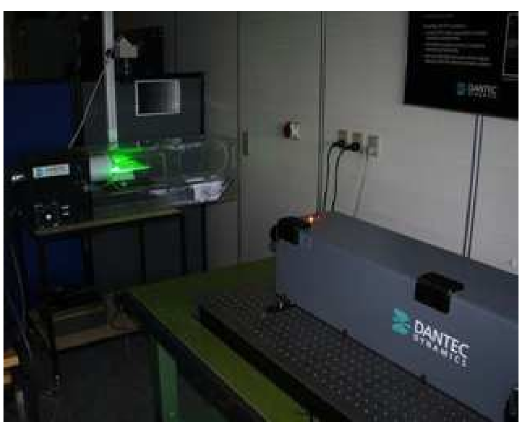


Figure 1: Sketch (a) and picture (b) of the experimental setup for PIV measurements of an unforced and forced jet.

The experimental setup is sketched and displayed in Figure 1. It consists of a jet emanating from a plenum by passing through a screen followed by a conical contraction. Inside the plenum a loudspeaker is installed that will imposed a time-harmonic acoustic signal on the jet. The PIV interrogation area starts 10 mm downstream of the nozzle exit and extends 54.7 mm in the streamwise and the transverse coordinate direction. The PIV images have been resolved on a 63×63 grid and are separated in time by 0.333 msec. The jet diameter is 10 mm and the center velocity of the jet is 7.5 m/sec for the unforced case, slightly less (7.4 m/sec) for the forced case. The forcing frequency has been chosen as 140 Hz.

The snapshots obtained from the PIV-measurements have been gathered and processed according to the DMD-algorithm outlined above. The eigenvalues of the low-dimensional system matrix **S** are displayed in Figure 2(a,b) for the unforced and the forced case, respectively.

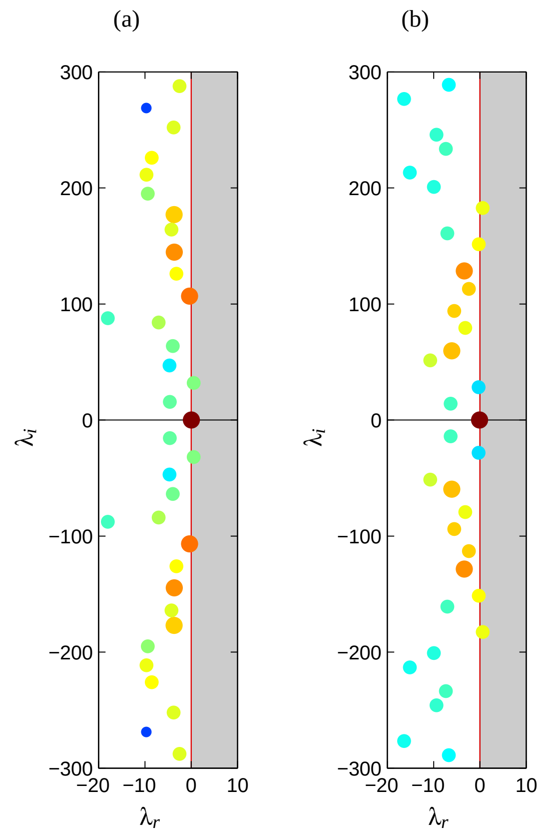


Figure 2: DMD-spectrum of (a) unforced and (b) forced jet for the parameters given in the text. The symbol size (large to small) and coloring (red to blue) of the eigenvalues indicate a coherence measure of the associated dynamic mode.

In both cases, an eigenvalue at the origin is observed, indicating the presence of a neutral and non-oscillatory structure. The associated dynamic mode (not shown) corresponds to the mean flow, i.e., the steady component in the snapshot sequence over the sampled time interval.

For the unforced case (Figure 2a) a dynamic mode with an inherent frequency of about 110 Hz appears to be dominant. Higher modes near the neutral line do not appear. For the forced case (Figure 2b) a different picture emerges: an eigenvalue with a frequency near the forcing frequency appears, together with a second (mildly unstable) mode at a slightly higher frequency. This seems to confirm that the jet is rather susceptible to external forcing by tuning into the frequency of the outside excitation.

The corresponding spatial structures of the inherent dynamics and the forced response are given in Figure 3 and 4, respectively. For the unforced case, a general tendency of a vortical structure developing in the downstream direction on the outer shear layer of the jet is observed; the perturbations identified by the DMD have little or negligible support near the jet exit. In addition, no precise streamwise scale can be detected, even though coherent

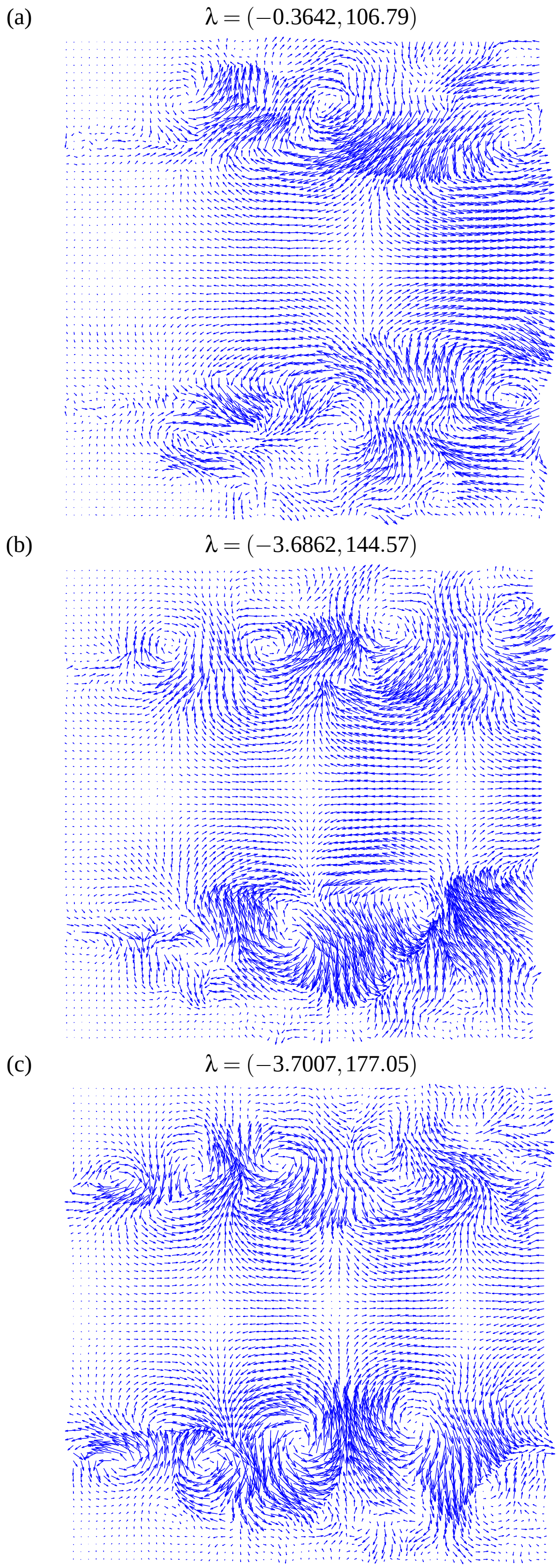


Figure 3: Three representative DMD-modes for the unforced jet, visualized by velocity vector plots.

vortices are present.

These facts should be contrasted with the forced case in which a marked response in terms of a locked-in spatial scale (see, in particular, Figure 4b) and a pronounced structure near the nozzle can be seen. It appears that the acoustic forcing of the jet has dramatically changed both the frequency and the shape of its most relevant dynamic mode. Flows that show a high susceptibility to external noise and excitation are often referred to as *amplifiers* and should be contrasted to flows

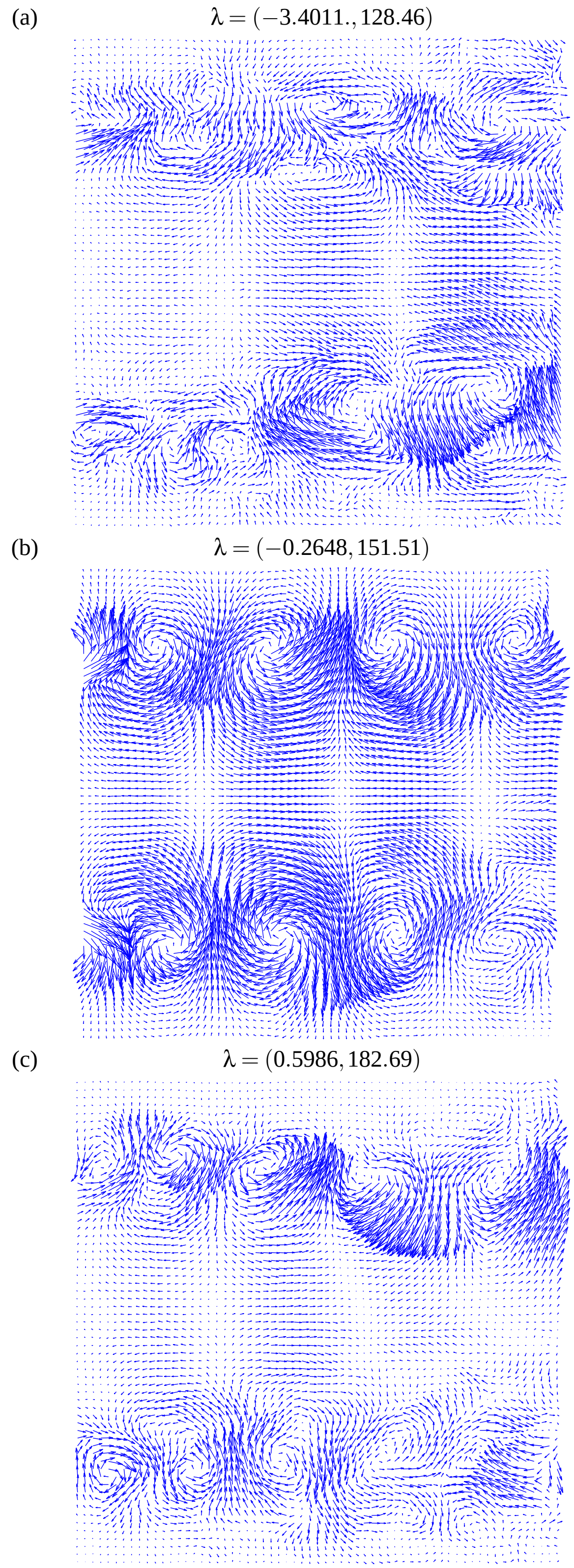


Figure 4: Three representative DMD-modes for the forced jet, visualized by velocity vector plots.

that act as *oscillators* (i.e., flows that display a robust lock-in to a self-sustained frequency relatively independent of the external excitation). Even though a distinction between these two flow cases has to be made by more rigorous arguments (see [3]), the response to external forcing, as displayed in the two DMD-analyses, can already provide a first indication.

4. CONCLUSIONS

The Dynamic Mode Decomposition (DMD) — a new tool of quantitative flow analysis based on the extraction of dynamically revelant structures from a time-resolved sequence of experimental (or numerical) flow fields — has been applied to the case of an unforced and forced jet. It has identified and quantified a distinct difference between the two cases as to the most relevant frequency and the spatial structure of the dynamically most prominent coherent structure. This study has also shown the value of temporal *and* spatial information in the description of flow behavior, and it is hoped that the Dynamic Mode Decomposition will become a useful tool in the quantitative analysis of fluid flows.

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